WESLEY COLLEGE
SOUTH PERTH

MATHEMATICS 3C/3D

Semester Two Examination 2012

Question/Answer Booklet

Section One: Calculator-free

Student Name: Solutions / Marking Guide

Teacher Name: ______________________

Time allowed for this section

Reading time before commencing work: Five (5) minutes
Working time for this section: Fifty (50) minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: *pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters*

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

Question 1 (2, 3, 2, 3 = 10 marks)

(a) Determine $\frac{dy}{dx}$ in each of the following (there is no need to simplify answers)

(i) $y = \frac{\sqrt{x^2 - 16}}{x}$

$$\frac{x \cdot \frac{1}{2}(x^2-16)^{-\frac{1}{2}} \cdot 2x - (x^2-16)^{\frac{1}{2}} \cdot 1}{x^2}$$

(ii) $y = \int_3^{e^{1-x}} \left(\frac{t-2}{t^2-1}\right) dt$

$$\left(\frac{e^{1-x}-2}{e^{2-2x}-1}\right) \cdot e^{1-x} \cdot (-1)$$

(b) (i) determine $\frac{d}{dx}\left(8x\, e^{4x}\right)$ $\quad 8x \cdot 4e^{4x} + 8e^{4x}$

(ii) hence or otherwise, determine $\int 32xe^{4x}\, dx$

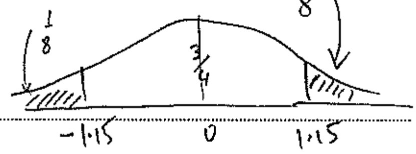
$$\int \frac{d}{dx}\left(8x\, e^{4x}\right) dx = \int 32x\, e^{4x} + \int 8e^{4x}\, dx$$

$$\therefore \int 32x\, e^{4x}\, dx = 8xe^{4x} - 2e^{4x} + c$$

Question 2 (1, 2, 2 = 5 marks)

A standard normal score of 1.15 is such that $P(z > 1.15) = 0.125$ *or* $\frac{1}{8}$

Use this information to determine:



(a) $P(-1.15 < z < 1.15)$

$= 3/4$

(b) $P(z < -1.15 \mid z < 1.15)$

$= \dfrac{P(z < -1.15)}{P(z < 1.15)}$

$= \dfrac{1/8}{7/8}$

$= 1/7$

(c) a 75% confidence interval (correct to one decimal place) for the mean of any sample of size 25 taken from a population of mean 50 and standard deviation 10.

$50 - 1.15 \times \frac{10}{5} \leq \mu \leq 50 + 1.15 \times \frac{10}{5}$

$50 - 2.3 \leq \mu \leq 50 + 2.3$

$47.7 \leq \mu \leq 52.3$

Question 3 **(2, 3 = 5 marks)**

A particle moves in a straight line so that its distance from a fixed point O is given by $x = \dfrac{4}{t+1}$ $t > 0$ $\rightarrow$ (Vel is never zero)

where x is the distance in metres at time t seconds.

(a) What is the average speed for the first two seconds?

$x(0) = 4$

$x(2) = 4/3$ $\Big\}$ $2\frac{2}{3}$ m $\therefore$ Av Speed $= \dfrac{2\frac{2}{3}}{2}$

$= 1\frac{1}{3}$ m/sec

(b) When does the particle reach a speed of $\frac{1}{4}$ m/sec

$\dot{x} = -\dfrac{4}{(t+1)^2}$ $\therefore$ Speed $= \dfrac{4}{(t+1)^2}$

$|\dot{x}| = \frac{1}{4} \Rightarrow \dfrac{4}{(t+1)^2} = \frac{1}{4} \Rightarrow t = 3$ sec

Question 4 **(2, 2 = 4 marks)**

The first stage of the 2013 AFL Training Camp for elite 18 year old footballers consists of three sets of skills testing A, B and C. From previous experience, the probabilities of success in skills A is 0.4, skills B is 0.6 and skills C is 0.75.

(a) For the top 50 18 year olds who attend the Training Camp, how many would be expected to succeed in all three skills tests?

$P(A \cap B \cap C) = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{4} = \frac{9}{50}$

$\therefore$ Expect 9 to succeed in all 3 skills

To be accepted into the second stage of the Training Program, a trainee must pass Test A and at least one of the other two tests.

(b) What is the probability that a person is accepted into the second stage?

$= 0.4 \times (1 - 0.4 \times 0.25)$

$= 0.4 \times 0.9$

$= 0.36$

Question 5 (5 marks)

Solve the inequality $\dfrac{x}{x-2} \leq \dfrac{8}{x}$

$x \neq 2$

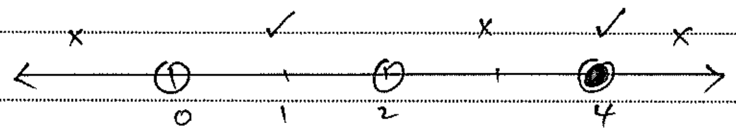
$x \neq 0$

$x^2 = 8x - 16$

$\therefore \quad x^2 - 8x + 16 = 0$

$(x-4)^2 = 0 \quad \Rightarrow \quad x = 4$

check :



$\therefore$ Sol^n $\quad 0 < x < 2 \quad \cup \quad x = 4$

Question 6 (3, 3 = 6 marks)

The probability distribution and cumulative probability distribution for a discrete random variable X are shown in the tables below:

x	0	1	2	3	4
$P(X = x)$	0.05	p	q	r	0.25

x	0	1	2	3	4
$P(X \leq x)$	0.05		0.55		1

and it is known that the expected value of the random variable is 2.5.

(a) Form three equations from the given information

$p + q + r = 0.7 \quad \ldots (1)$

$p + 2q + 3r + 1 = 2.5 \quad \Rightarrow \quad p + 2q + 3r = 1.5 \quad \ldots (2)$

$p + q + 0.05 = 0.55 \quad \Rightarrow \quad p + q = 0.5 \quad \ldots (3)$

(b) Solve for p, q and r Eqn ①-③ $\}$ $r = 0.2$

then $p + 2q + 0.6 = 1.5 \Rightarrow$ $\left.\begin{array}{l} p+2q = 0.9 \\ p+q = 0.5 \end{array}\right\} q = 0.4$

$\therefore$ $p = 0.1$, $q = 0.4$, $r = 0.2$

Question 7 (1, 3, 3, 2 = 9 marks)

(a) Functions f, g, h are defined:

$f(x) = \sqrt{4-x}$ $\quad g(x) = 2^x - 12$ $\quad h(x) = e^{2x-1} - 1$ $\quad$ Determine:

(i) $f o g(x)$

$= \sqrt{4-(2^x-12)}$

$= \sqrt{16-2^x}$

(ii) the domain and range of $f o g(x)$

$D_x: \; x \leq 4$

$R_y: \; 0 \leq y < 4$

(iii) $j(x)$ if $j o f(x) = 1-x$

$j(\sqrt{4-x}) = 1 - x$ $\quad\quad u = \sqrt{4-x}$

$j(u) = 1-(4-u^2)$ $\quad\quad u^2 = 4-x$

$\therefore j(x) = x^2 - 3$ $\quad\quad x = 4-u^2$

(b) The function $k(x)$ is a transformation of $h(x)$ consisting of a horizontal dilation of factor 2 followed by a vertical translation of 1 unit. Write down the equation of $k(x)$ in its simplest form.

$h(x) = e^{2(x-\frac{1}{2})} - 1$

$\hookrightarrow$ $k(x) = e^{\frac{1}{2}(2(x-\frac{1}{2}))} - 1 + 1$

$\Rightarrow k(x) = e^{x-\frac{1}{2}}$

Question 8 (6 marks)

A cubic function $f(x)$ whose derivative $f'(x)$ is given by $f'(x) = 2x^2 - 5x + 2$ has a local or relative minimum value of 2. Determine the local or relative maximum value.

$f''(x) = 4x - 5$

$f'(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$

i.e. $(2x-1)(x-2) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$

$f''\left(\frac{1}{2}\right) = -3$ $\therefore$ MAX

$f''(2) = 3$ $\therefore$ min

$\therefore$ $f(2)$ produces the min value of 2

$f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 2x + c$

$2 = 2\times\frac{8}{3} - 5\times\frac{4}{2} + 4 + c$

$c = 8 - \frac{16}{3} \Rightarrow c = \frac{8}{3}$

$\therefore f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 2x + \frac{8}{3}$

$f\left(\frac{1}{2}\right) = \frac{1}{12} - \frac{5}{8} + 1 + \frac{8}{3}$

$= \frac{2 - 15 + 24 + 64}{24}$

$= \frac{75}{24}$

END OF SECTION 1